

Ch 10 Exponential and Logarithmic Functions

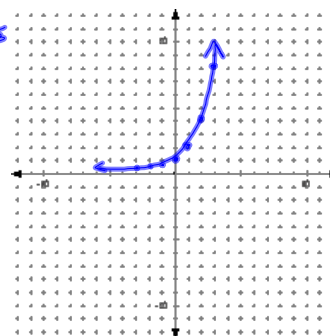
10-1 Exponential Functions

Exponential function--function with a variable in the exponent

ex

$$y = 1.2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



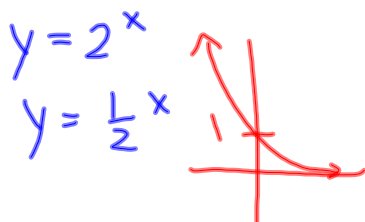
Form

$$y = a b^x$$

$$a \neq 0$$

$$b > 0$$

$$b \neq 1$$



graph on calculator

Characteristics (p524)

1. The function is continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x -axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y -intercept is a .
6. The graphs of $y = ab^x$ and $y = a\left(\frac{1}{b}\right)^x$ are reflections across the y -axis.

Write an exponential function whose graph passes through the given points.

 $(0, 5)$ $(2, 45)$

$$y = a \cdot b^x$$

$$a = 5$$

$$y = 5 \cdot b^x$$

$$45 = 5 \cdot b^2$$

$$9 = b^2$$

$$\pm 3 = b$$

$$b = 3$$

$$y = 5 \cdot 3^x$$

Write an exponential function whose graph passes through the given points.

 $(0, 4)$ $(3, \frac{1}{2})$

$$y = 4 \cdot b^x$$

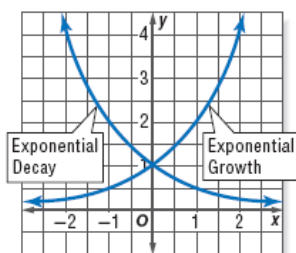
$$\frac{1}{2} = 4 \cdot b^3$$

$$\left(\frac{1}{8}\right)^{\frac{1}{3}} = \left(b^3\right)^{\frac{1}{3}}$$

$$\frac{1}{2} = b$$

$$y = 4\left(\frac{1}{2}\right)^x$$

There are two types of exponential functions: **exponential growth** and **exponential decay**. The base of an exponential growth function is a number greater than one. The base of an exponential decay function is a number between 0 and 1.

**Key Concept****Exponential Growth and Decay**

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

ex

$$5^{4-t} = 25^{t-1}$$

Same base

$$5^{4-t} = (5^2)^{t-1}$$

$$4-t = 2t-2 \quad 5^{4-2} = 25^{2-1}$$

$$6 = 3t$$

$$2 = t$$

Must Check!

Solving exponential equations.

ex

$$8^x = \frac{1}{4}$$

$$2^{3x} = 2^{-2}$$

$$8^{-2/3} = \frac{1}{4} \checkmark$$

$$2^{-2} = \frac{1}{4} \checkmark$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

ex

$$\sqrt[3]{32} = 2^x$$

$$\sqrt[3]{2^5}$$

$$2^{5/3} = 2^x$$

$$\frac{5}{3} = x$$

Check

$$2^{5/3}$$

$$2^5 \sqrt[3]{32}$$

ex

$$9^{x-3} = 27$$

$$(3^2)^{x-3} = 3^3$$

$$2x-6 = 3$$

$$x = 4.5$$

Check

$$9^{4.5-3} \stackrel{?}{=} 27$$

$$9^{3/2}$$

$$3^3 = 27 \checkmark$$

Do:

1. $4^{x-2} = 64^x$

2. $4^{2x+5} = 16^{x+1}$

Exponential Inequalities

ex

$3^x < \frac{1}{27}$

$3^x < 3^{-3}$

$x < -3$

ex

$$5^x > \sqrt{125}$$

$$5^x > \sqrt{5^3}$$

$$5^x > 5^{3/2}$$

$$x > \frac{3}{2}$$

ex

$25^{2x} < 5^{(x+6)}$

$(5^2)^{2x} < 5^{(x+6)}$

$4x < x+6$

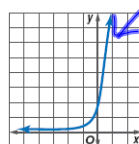
$3x < 6$
 $x < 2$

1. **OPEN ENDED** Give an example of a value of b for which $y = b^x$ represents exponential decay.
2. **Identify** each function as linear, quadratic, or exponential.
- a. $y = 3x^2$ b. $y = 4(3)^x$ c. $y = 2x + 4$ d. $y = 4(0.2)^x + 1$

Match each function with its graph.

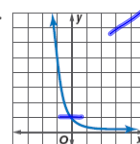
3. $y = 5^x$

a.



4. $y = 2(5)^x$

b.



5. $y = \left(\frac{1}{5}\right)^x$

c.

